WELLS

OUTLINE:

- Types/uses of wells
- Well construction
- Flow to wells
  - steady & transient flow
  - aquifer testing & hydraulic control
  - principle of superposition

PURPOSE:

1. Wells are our tool to observe/control groundwater systems

2. Illustrate principles of saturated flow with field examples

Types & Uses of Wells:

Type	**Use**	**Diameter**	**Screen Length**
Piezometer	measuring h	1 - 2 inches	Short
Monitor Well	water sample analysis	2 - 4 inches	cm (MLS) to 10 ft.
Pumping Well	aquifer testing	6 inch +	zone of interest
	hydraulic control	6 inch +	
	water production	8 - 24 in.	Long (> 100ft)

WELL CONSTRUCTION (see CH: 8; Bedient et al.: ch. 5)

1. Construction Methods:

Hollow-stem auger (continuous flights)
- up to ~ 150 ft.
- allows collection of soil samples
- preferred method for site investigations
- unconsolidated soils only

Solid-stem auger
- unconsolidated soils only

Cable tool
- good for observing cuttings
- up to ~ 150 ft

Rotary Methods (air/mud/water)
- good for depth
- mud necessary for heaving sands

Driving (shallow wells)
- no cuttings but cheap

Jetting
- high pressure water washes out aquifer material

2. Well schematic

3. Well development

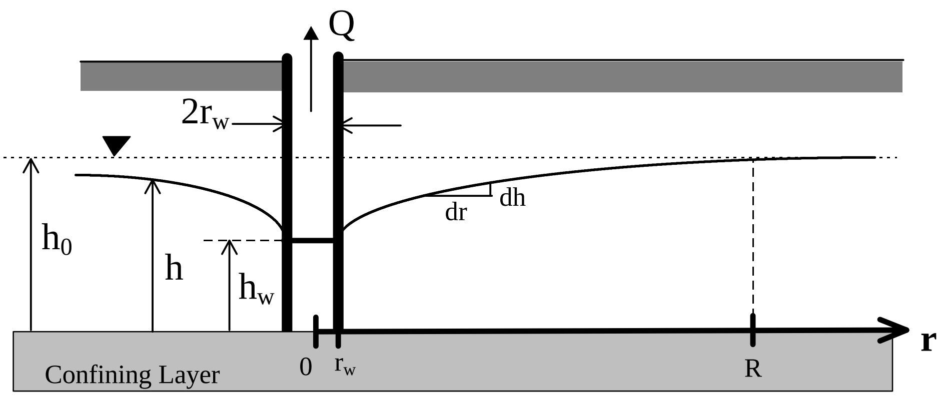
Purpose:
- provide sand-free well @ max. specific capacity
- repair damage to aquifer
- prevent fine particles from entering the well

Methods:
- surge water (move cylinder up/down well)
- add water down well/through screen/up borehole
- over pumping

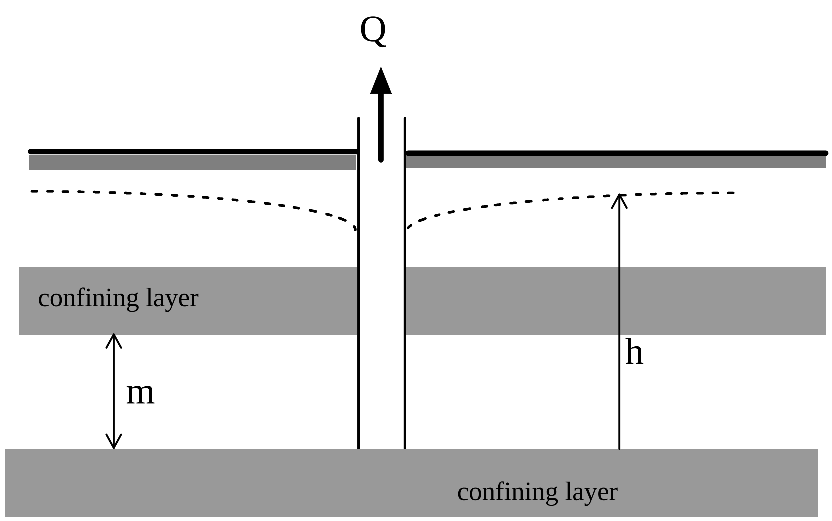
Explore solving the flow equation:

2 Scenarios involving radial flow to a well:

Unconfined Aquifer:



Confined Aquifer

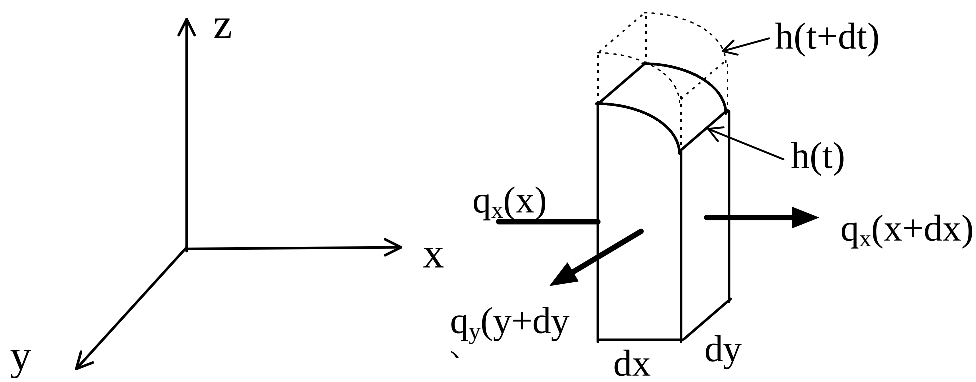


RADIAL FLOW TO WELLS:

- Work through equations for unconfined aquifer
- Show solution for confined aquifer

Consider 2-D flow in porous media:
Incompressible fluid
Nondeformable media

Continuity Equation: Δ Storage = Flow in - Flow out



$$\Delta\,\text{Storage} = n\frac{\partial h}{\partial t}\,dx\,dy$$

Flow in = $q_x\,h\,dy + q_y\,h\,dx$

$$\text{Flow out} = q_x\,h\,dy + \frac{\partial}{\partial x}\left(q_x h\,dy\right)dx + q_y\,h\,dx + \frac{\partial}{\partial y}\left(q_y h\,dx\right)dy$$

Net continuity equation:

$$n\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left(q_x h\right) - \frac{\partial}{\partial y}\left(q_y h\right)$$

Insert Darcy Equation:

$$n\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(h\,K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\,K_y\frac{\partial h}{\partial y}\right)$$

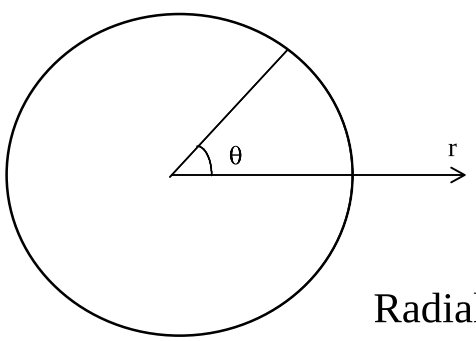
Assume: Homogeneous/Isotropic Media

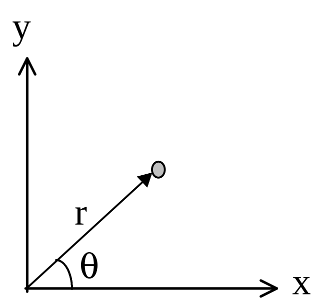
$$n\frac{\partial h}{\partial t} = K\left(\frac{\partial}{\partial x}h\frac{\partial h}{\partial x} + \frac{\partial}{\partial y}h\frac{\partial h}{\partial y}\right)$$

Rewrite in terms of h^2: $h\frac{dh}{dx} = \frac{1}{2}\frac{dh^2}{dx}$

$$n\frac{\partial h}{\partial t} = \frac{K}{2}\left(\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2}\right)$$

Rewrite the equations in radial coordinates:





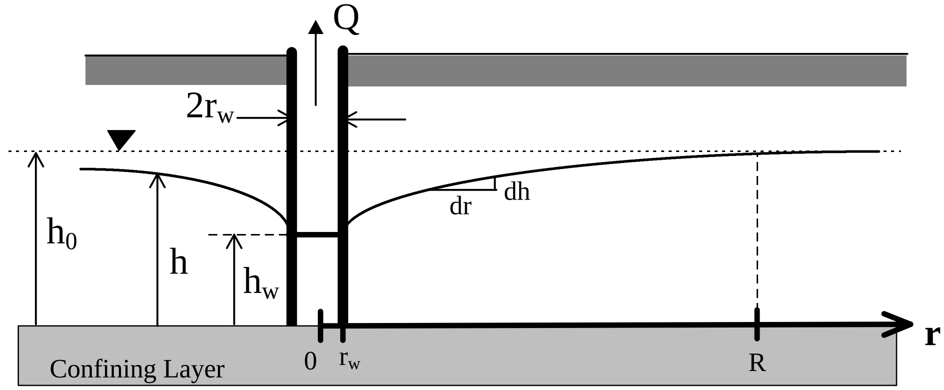
Radial coordinates:
$h(x,y) \rightarrow h(r,\theta) \rightarrow h(r)$ [isotropic]
$r = (x^2 + y^2)^{1/2}$

using chain rule for differential equations:
(i.e.: $\frac{\partial h}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial h}{\partial r}$*)*

$$n\frac{\partial h}{\partial t} = \frac{K}{2}\left(\frac{\partial^2 h^2}{\partial r^2} + \frac{1}{r}\frac{\partial h^2}{\partial r}\right)$$

Governing Equation: Transient flow to a well in a water table aquifer

Assumed: Radial flow (1-D)
Homogeneous, Isotropic conditions
Incompressible fluid/Nondeformable media
Media drains completely (if not; replace n with S_y)



Governing equation for confined aquifer:

$$mS_s\frac{\partial h}{\partial t} = K\,m\left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right)$$

To solve the equation for hydraulic head:

1. Do we want:
   - transient solution [$h(r,t)$] ?
   - steady state solution [$h(r)$]?

2. Rewrite governing equation in terms of h using:
   - analytical or numerical solution, and
   - appropriate boundary/initial conditions.

Examine Boundary Conditions:

- Initial conditions: $h(r, t=0) = h_0$

- Boundary Condition 1: $h(r=\text{¥}\ , t) = h_0$

- Boundary Condition 2: $$r\frac{\partial h^2}{\partial r}\bigg|_{(r=r_w,t>0)} = \frac{Q}{\pi K}$$

  if pumping rate, Q, is constant: $Q = K\frac{\partial h}{\partial r}(2\pi rh)$

**** Similar to the IC and BC's used by Theis for transient flow to a well in a confined aquifer******

What would be the difference for the confined case?

BC2: : $$r\frac{\partial h}{\partial r}\bigg|_{(r=r_w,t>0)} = \frac{Q}{2\pi Km}$$ m = aquifer thickness

Flow from well in confined aquifer, $Q = K\frac{\partial h}{\partial r}(2\pi rm)$

Alternative choices of Boundary Conditions:

- Boundary Condition 1a: $h(r=R, t) = h_0$

where R = radius of influence of well
= distance at which drawdown is effectively zero
= (can't measure drawdown ≤ 0.01 ft).

How to calculate R?
Empirical relationships: $R = 3000\,(h_0 - h_w)\,K^{0.5}$
(R, h in m, K in m/s)

- Boundary Condition 2a: $h(r=r_w) = h_w$ (at steady state only)

Replace constant flux boundary with constant head boundary

Solve Equation for STEADY STATE:

Governing Equation: $$0 = \frac{K}{2}\left(\frac{\partial^2 h^2}{\partial r^2} + \frac{1}{r}\frac{\partial h^2}{\partial r}\right)$$

Can be rewritten: $$0 = \frac{d}{dr}\left(rh\frac{dh}{dr}\right)$$

which is a second order ordinary differential equation which has an exact analytical solution.

Use: BC 1a: $h(r=R) = h_0$
BC 2a: $h(r=r_w) = h_w$

Analytical solution to governing equation:

$h = f(r, K_1, K_2)$

Use B.C.'s to solve for K_1, K_2 in terms of R, r_w, h_0, and h_w.

Steady-State Solution in terms of *h*:

$$h^2 - h_w^2 = \frac{\left(h_0^2 - h_w^2\right)}{\ln\left(R/r_w\right)} \ln\left(r/r_w\right)$$

- Solve for hydraulic head at any point.

Steady-State Solution in terms of well discharge, *Q*:

$$Q = qA = K\frac{dh}{dr}(2\pi rh) = \pi Kr\left(\frac{dh^2}{dr}\right)$$

$$Q = \frac{\pi K\left(h_0^2 - h_w^2\right)}{\ln\left(R/r_w\right)}$$

Steady-state well discharge equation, unconfined aquifer

- Solve for pumping rate necessary to acheive a given drawdown.

$$Q = \frac{2\pi Km\left(h_0 - h_w\right)}{\ln\left(R/r_w\right)}$$

Steady-state well discharge equation, confined aquifer

Solve Equation for TRANSIENT CONDITIONS:

3 choices:
1. Numerical approach
2. Theis solution
3. Laplace transformation (?)

Numerical Approach:

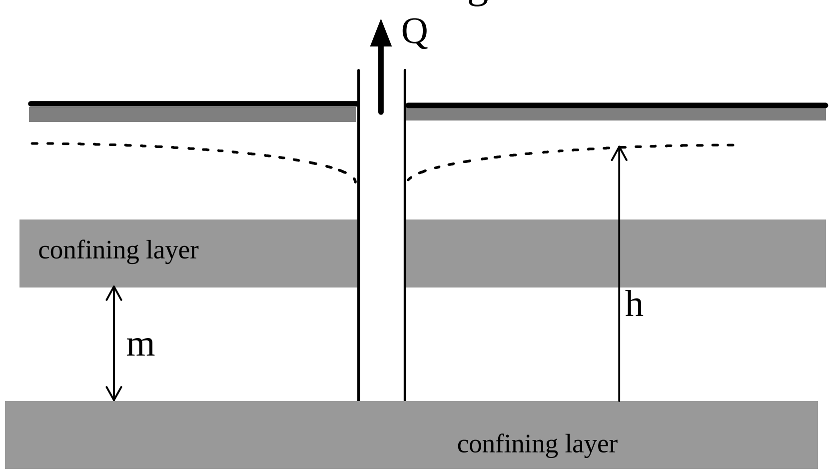
Rewrite Governing Equation using numerical notation:

$$n\frac{\left[h_{r,t+\Delta t}-h_{r,t-\Delta t}\right]}{2\Delta t}=\frac{K}{2}\left(\frac{\left[h_{r-\Delta r,t}{}^{2}+h_{r+\Delta r,t}{}^{2}-2h_{r,t}{}^{2}\right]}{(\Delta r)^{2}}+\frac{1}{r}\frac{\left[h_{r+\Delta r,t}{}^{2}-h_{r-\Delta r,t}{}^{2}\right]}{2\Delta r}\right)$$

Solve for $h_{r,t+Dt}$ using I.C., B.C.'s 1a and 2.

Theis Solution:

The Theis equation solves transient flow to a well in a confined aquifer--flow area does not change with time.



Theis Solution (continued):

Governing equation is f(h), not f(h^2).

Use I.C.; B.C. 1 and 2.

- Note, B.C. 2 is: $r\left.\frac{\partial h}{\partial r}\right|_{r=r_w} = \frac{Q}{2\pi mK},\ r_w \to 0, t > 0$

results in:

$$s = \frac{Q}{4\pi mK} W(u)$$

where: *s* = drawdown at well, h_0 - h_w
W(u) = well function of *u*

- Okay for unconfined aquifer if $s << m$
- See AH: Ch. 7.4; Domenico & Schwartz: Ch. 5.2 for more details

3. Laplace Transformation:

1. Convert governing equation to Laplace space (removes $\frac{\partial h}{\partial t}$ term)
2. Find analytical solution in Laplace space to ordinary differential equation (use I.C., B.C.'s as appropriate).
3. Convert solution back to real space (can be done numerically).

Advantages: Possibly use better B.C. (don't estimate *R*)
Not confined to Theis solution limitations.
Possibly faster than numerical approach.

Disadvantage: Difficulty finding analytical solution.

Now we have steady state and transient solution methods.

What do we do with them?

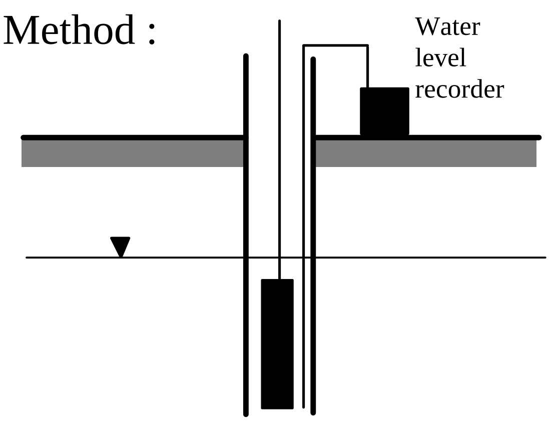
1. Aquifer parameter estimation:
   well flow tests to determine *K*
2. Determine hydraulic head changes in response to pumping
   hydraulic control capture zones

Determining Hydraulic Conductivity:

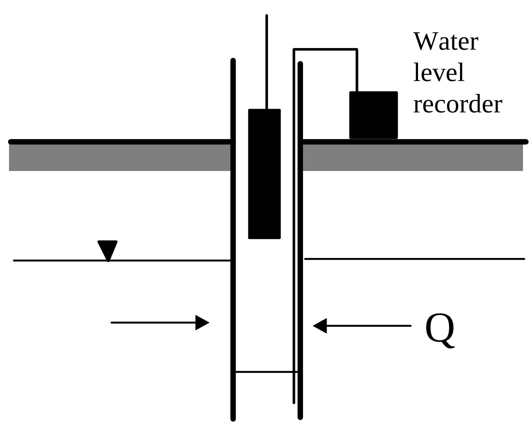
Need to estimate hydraulic conductivity, *K*:
1. Lab: column tests (permeameter [Darcy's experiment])
2. Field: slug tests (transient well flow problem)
3. Field: pump tests (transient or steady state well flow problem)

2. Slug test:

Method :



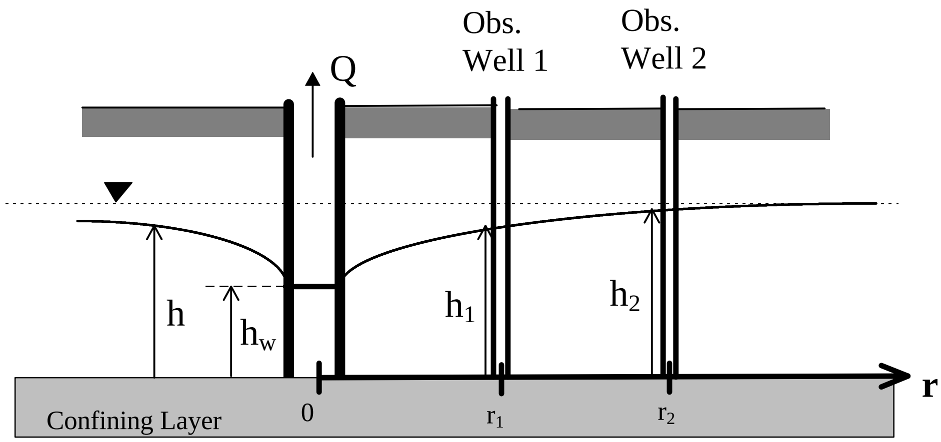
- Displace water in well with slug.
- Allow water level to equilibrate.



- Remove slug from well quickly.
- Record water level with time as water flows into the well.

- What governing equation/boundary conditions must be used?
- What are the limitations of this method?

3. Pump test:



Draw down at well 1, $s_1 = h_0 - h_1$

Theim Equation (unconfined aquifer):

$$s_2^{\,2} - s_1^{\,2} - 2h_0(s_2 - s_1) = \frac{Q}{\pi K}\ln\left(\frac{r_2}{r_1}\right)$$

- Where did this equation come from?
- What are the boundary conditions?
- How do we conduct the pump test to determine *K*?
- What is the advantage of the pump test over the slug test?
- How can we investigate anisotropic properties of *K*?
- How can we investigate uniformity properties of *K*?

Solution for confined aquifer:

$$s_1 - s_2 = \frac{Q}{2\pi K m}\ln\left(\frac{r_2}{r_1}\right)$$